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Violation of Time Reversal Invariance in the Decays $K_L \to \pi^+\pi^-\gamma$ and $K_L \to \pi^+\pi^- e^+ e^-$

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The origin of the large CP-odd and T-odd asymmetry observed in the decay $K_L \to \pi^+\pi^- e^+ e^-$ is traced to the polarization properties of the photon in the decay $K_L \to \pi^+\pi^-\gamma$. The Stokes vector of the photon $\vec{S} = (S_1, S_2, S_3)$ is studied as a function of the photon energy and found to possess CP-violating components S_1 and S_2 which are sizeable over a large part of the phase space, despite being proportional to the ϵ parameter of the K_L wave function. The component S_2 is T-even and manifests itself as a circular polarization of the photon, while S_1 is T-odd and gives rise to the asymmetry observed in $K_L \to \pi^+\pi^- e^+ e^-$. The latter is shown to survive in the "hermitian" limit in which all unitarity phases are absent, and represents a genuine example of time reversal symmetry breaking in a CPT invariant theory.

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The KTeV experiment has reported the observation of a large CP-violating, T-odd asymmetry in the decay $K_L \to \pi^+\pi^- e^+ e^-$ [1], in agreement with a theoretical prediction made some years ago [2,3]. In this letter, we trace the origin of this effect to a large violation of CP-invariance and T-invariance in the decay $K_L \to \pi^+\pi^-\gamma$, which is hidden in the polarization state of the photon. We explain why the effect is large, despite the fact that it stems entirely from the ϵ-impurity of the K_L wave function. Our analysis demonstrates that the T-odd asymmetry does not vanish in the limit in which unitarity phases, expressing the non-hermiticity of the effective Hamiltonian, are switched off, and thus represents a genuine example of time reversal non-invariance.

The decay $K_L \to \pi^+\pi^-\gamma$ is known empirically [4] to contain a bremsstrahlung component (IB) as well as a direct emisson component (DE), with a relative strength $DE/(DE + IB) = 0.68$ for photons above $20\,MeV$. By contrast, the decay $K_S \to \pi^+\pi^-\gamma$ is well reproduced by pure bremsstrahlung. The simplest matrix element consistent with these features is [2]

$$\begin{aligned}\mathcal{M}(K_S \to \pi^+\pi^-\gamma) &= e f_S \left[\frac{\epsilon \cdot p_+}{k \cdot p_+} - \frac{\epsilon \cdot p_-}{k \cdot p_-}\right] \\ \mathcal{M}(K_L \to \pi^+\pi^-\gamma) &= e f_L \left[\frac{\epsilon \cdot p_+}{k \cdot p_+} - \frac{\epsilon \cdot p_-}{k \cdot p_-}\right] \\ &\quad + e\frac{f_{DE}}{{M_K}^4}\epsilon_{\mu\nu\rho\sigma}\epsilon^\mu k^\nu {p_+}^\rho {p_-}^\sigma\end{aligned} \tag{1}$$

where

$$\begin{aligned} f_L &\equiv |f_S| g_{Br},\; g_{Br} = \eta_{+-} e^{i\delta_0(s={M_K}^2)}, \\ f_{DE} &\equiv |f_S| g_{M1},\; g_{M1} = i(0.76)e^{i\delta_1(s)}. \end{aligned} \tag{2}$$

Here the direct emission has been represented by a CP-conserving magnetic dipole coupling g_{M1}, whose magnitude $|g_{M1}| = 0.76$ is fixed by the empirical ratio DE/IB. The phase factors appearing in g_{Br} and g_{M1} are dictated by the Low theorem for bremsstrahlung, and the Watson theorem for final state interactions. The factor i in g_{M1} is a consequence of CPT invariance [5]. The matrix element for $K_L \to \pi^+\pi^-\gamma$ contains simultaneously electric multipoles associated with bremsstrahlung ($E1$, $E3$, $E5$, $\cdots$), which have $CP = +1$, and a magnetic $M1$ multipole with $CP = -1$. It follows that interference of the electric and magnetic emissions should give rise to CP-violation.

To determine the nature of this interference, we write the $K_L \to \pi^+\pi^-\gamma$ amplitude more generally as

$$\begin{aligned}\mathcal{M}(K_L \to \pi^+\pi^-\gamma) = \frac{1}{{M_K}^3}\{&E(\omega, \cos\theta) \\ &\times [\epsilon \cdot p_+\, k \cdot p_- - \epsilon \cdot p_-\, k \cdot p_+] \\ &+ M(\omega, \cos\theta)\epsilon_{\mu\nu\rho\sigma}\epsilon^\mu k^\nu {p_+}^\rho {p_-}^\sigma\}\end{aligned} \tag{3}$$

where ω is the photon energy in the K_L rest frame, and θ is the angle between π^+ and γ in the $\pi^+\pi^-$ rest frame. In the model represented by Eqs. (1) and (2), the electric and magnetic amplitudes are (omitting a common factor $e|f_S|/M_K$)

$$\begin{aligned} E &= \left(\frac{2M_K}{\omega}\right)^2 \frac{g_{Br}}{1 - \beta^2\cos^2\theta} \\ M &= g_{M1} \end{aligned} \tag{4}$$

where $\beta = (1 - 4{m_\pi}^2/s)^{1/2}$, $\sqrt{s}$ being the $\pi^+\pi^-$ invariant mass. The Dalitz plot density, summed over photon polarizations is

$$\begin{aligned}\frac{d\Gamma}{d\omega\, d\cos\theta} = \frac{1}{512\pi^3}\left(\frac{\omega}{M_K}\right)^3 \beta^3 \left(1 - \frac{2\omega}{M_K}\right) \\ \times \sin^2\theta\left[|E|^2 + |M|^2\right].\end{aligned} \tag{5}$$

Clearly, there is no interference between the electric and magnetic multipoles if the photon polarization is unobserved. Therefore, any CP-violation involving the interference of g_{Br} and g_{M1} is encoded in the polarization state of the photon.

The photon polarization can be defined in terms of the density matrix

$$\rho = \begin{pmatrix} |E|^2 & E^*M \\ EM^* & |M|^2 \end{pmatrix} = \frac{1}{2}\left(|E|^2 + |M|^2\right)\left[\mathbb{1} + \vec{S}\cdot\vec{\tau}\right] \quad (6)$$

where $\vec{\tau} = (\tau_1,\, \tau_2,\, \tau_3)$ denotes the Pauli matrices, and $\vec{S}$ is the Stokes vector of the photon with components

$$\begin{aligned} S_1 &= 2Re\,(E^*M)\,/\left(|E|^2 + |M|^2\right) \\ S_2 &= 2Im\,(E^*M)\,/\left(|E|^2 + |M|^2\right) \\ S_3 &= \left(|E|^2 - |M|^2\right)/\left(|E|^2 + |M|^2\right). \end{aligned} \quad (7)$$

The component S_3 measures the relative strength of the electric and magnetic radiation at a given point in the Dalitz plot. The effects of CP-violation reside in the components S_1 and S_2, which are proportional to $Re\,(g_{Br}{}^* g_{M1})$ and $Im\,(g_{Br}{}^* g_{M1})$, respectively. Physically, S_2 is the net circular polarization of the photon: it is proportional to the difference of $|E - iM|^2$ and $|E + iM|^2$, which are the probabilities for left-handed and right-handed radiation. Such a polarization is a CP-odd, T-even effect, which is known to be possible in decays like $K_L \to \pi^+\pi^-\gamma$ or $K_{L,S} \to \gamma\gamma$ whenever there is CP-violation accompanied by unitarity phases [5,6]. To understand the significance of S_1, we examine the dependence of the $K_L \to \pi^+\pi^-\gamma$ decay on the angle ϕ between the polarization vector $\vec{\epsilon}$ and the unit vector $\vec{n}_\pi$ normal to the decay plane (we choose coordinates such that $\vec{k} = (0,\, 0,\, k)$, $\vec{n}_\pi = (1,\, 0,\, 0)$, $\vec{p}_+ = (0,\, p\sin\theta,\, p\cos\theta)$ and $\vec{\epsilon} = (\cos\phi,\, \sin\phi,\, 0)$):

$$\begin{aligned} \frac{d\Gamma}{d\omega\, d\cos\theta\, d\phi} &\sim |E\sin\phi - M\cos\phi|^2 \\ &\sim 1 - \left[S_3\cos 2\phi + S_1\sin 2\phi\right]. \end{aligned} \quad (8)$$

Notice that the Stokes parameter S_1 appears as a coefficient of a term $\sin 2\phi$ which changes sign under CP as well as T. Thus S_1 is a measure of a CP-odd, T-odd correlation. The essential idea of Refs. [2,3] is to use in place of $\vec{\epsilon}$, the vector $\vec{n}_l$ normal to the plane of the Dalitz pair in the reaction $K_L \to \pi^+\pi^-\gamma^* \to \pi^+\pi^-e^+e^-$. This motivates the study of the distribution $d\Gamma/d\phi$ in the decay $K_L \to \pi^+\pi^-e^+e^-$, where ϕ is the angle between the $\pi^+\pi^-$ and e^+e^- planes.

To obtain a quantitative idea of the magnitude of CP-violation in $K_L \to \pi^+\pi^-\gamma$, we show in Fig. 1a the three components of the Stokes vector as a function of the photon energy. These are calculated from the amplitudes (4) using weighted averages of $|E|^2$, $|M|^2$, E^*M and EM^* over $\cos\theta$ [7]. The values of S_1 and S_2 are remarkably large, considering that the only assumed source of CP-violation is the ϵ-impurity in the K_L wave-function ($\epsilon = \eta_{+-}$). Clearly the factor $(2M_K/\omega)^2$ in E enhances it to a level that makes it comparable to the CP-conserving amplitude M. This is evident from the behaviour of the parameter S_3, which swings from a dominant electric behaviour at low ω ($S_3 \approx 1$) to a dominant magnetic behaviour at large ω ($S_3 \approx -1$), with a zero in the region $\omega \approx 60\,MeV$. The essential difference between the T-odd parameter S_1 and the T-even parameter S_2 comes to light when we compare their behaviour in the "hermitian" limit: this is the limit in which the T-matrix or effective Hamiltonian governing the decay $K_L \to \pi^+\pi^-\gamma$ is taken to be hermitian, all unitarity phases related to real intermediate states being dropped. This limit is realized by taking $\delta_0,\, \delta_1 \to 0$, and $arg\,\epsilon \to \pi/2$. The last of these follows from the fact that ϵ may be written as

$$\epsilon = \frac{\Gamma_{12} - \Gamma_{21} + i\,(M_{12} - M_{21})}{\gamma_S - \gamma_L - 2i\,(m_L - m_S)} \quad (9)$$

where $H_{eff} = M - i\Gamma$ is the mass matrix of the K^0-$\overline{K}^0$ system. The hermitian limit obtains when $\Gamma_{12} = \Gamma_{21} = \gamma_S = \gamma_L = 0$. As seen from Fig. 1b, S_2 vanishes in this limit, but S_1 survives, as befits a CP-odd, T-odd observable. This difference in behaviour is obvious from the fact that in the hermitian limit

$$\begin{aligned} S_1 &\sim Re(g_{Br}{}^* g_{M1}) \sim \sin(\phi_{+-} + \delta_0 - \delta_1) \to 1 \\ S_2 &\sim Im(g_{Br}{}^* g_{M1}) \sim \cos(\phi_{+-} + \delta_0 - \delta_1) \to 0 \end{aligned} \quad (10)$$

Fig. 1c shows what happens in the CP-invariant limit $\epsilon \to 0$: the parameters S_1, S_2 collapse to zero, while S_3 attains the uniform value -1. It is clear that we are dealing here with an exceptional situation in which a CP-impurity of a few parts in a thousand in the K_L wave-function is magnified into a huge CP-odd, T-odd effect in the photon polarization.

We can now examine how these large CP-violating effects are transported to the decay $K_L \to \pi^+\pi^-e^+e^-$. The matrix element for $K_L \to \pi^+\pi^-e^+e^-$ can be written as [2,3]

$$\begin{aligned} \mathcal{M}(K_L \to \pi^+\pi^-e^+e^-) = \mathcal{M}_{br} &+ \mathcal{M}_{mag} \\ &+ \mathcal{M}_{CR} + \mathcal{M}_{SD}. \end{aligned} \quad (11)$$

Here $\mathcal{M}_{br}$ and $\mathcal{M}_{mag}$ are the conversion amplitudes associated with the bremsstrahlung and $M1$ parts of the $K_L \to \pi^+\pi^-\gamma$ amplitude. In addition, we have introduced an amplitude $\mathcal{M}_{CR}$ denoting $\pi^+\pi^-$ production in a $J = 0$ state (not possible in a real radiative decay), as well as an amplitude $\mathcal{M}_{SD}$ associated with the short-distance interaction $s \to d\,e^+e^-$. The last of these turns out to be numerically negligible because of the smallness of the CKM factor $V_{ts}V_{td}{}^*$. The s-wave amplitude $\mathcal{M}_{CR}$, if approximated by the K^0 charge radius diagram, makes a small ($\sim 1\%$) contribution to the decay rate. Thus the dominant features of the decay are due to the conversion amplitude $\mathcal{M}_{br} + \mathcal{M}_{mag}$.

Within such a model, one can calculate the differential decay rate in the form [3]

$$d\Gamma = I(s_\pi,\, s_l,\, \cos\theta_l,\, \cos\theta_\pi,\, \phi)\, ds_\pi\, ds_l\, d\cos\theta_l\, d\cos\theta_\pi\, d\phi. \quad (12)$$

Here s_π (s_l) is the invariant mass of the pion (lepton) pair, and θ_π (θ_l) is the angle of the π^+ (l^+) in the $\pi^+\pi^-$ (l^+l^-) rest frame, relative to the dilepton (dipion) momentum vector in that frame. The all-important variable ϕ is defined in terms of unit vectors constructed from the pion momenta $\vec{p_\pm}$ and lepton momenta $\vec{k_\pm}$ in the K_L rest frame:

$$
\begin{aligned}
\vec{n}_\pi &= (\vec{p}_+ \times \vec{p}_-) \,/\, |\vec{p}_+ \times \vec{p}_-| \,, \\
\vec{n}_l &= \left(\vec{k}_+ \times \vec{k}_-\right) / \left|\vec{k}_+ \times \vec{k}_-\right| , \\
\vec{z} &= (\vec{p}_+ + \vec{p}_-) \,/\, |\vec{p}_+ + \vec{p}_-| \,,
\end{aligned}
$$

$$
\begin{aligned}
\sin\phi \, &= \vec{n}_\pi \times \vec{n}_l \cdot \vec{z} \;\; (CP=-, T=-), \\
\cos\phi \, &= \quad \vec{n}_\pi \cdot \vec{n}_l \quad\; (CP=+, T=+).
\end{aligned}
\tag{13}
$$

In Ref. [2], an analytic expression was derived for the 3-dimensional distribution $d\Gamma/ds_l\,ds_\pi\,d\phi$, which has been used in the Monte Carlo simulation of this decay. In Ref. [3], a formalism was presented for obtaining the fully differential decay function $I(s_\pi,\, s_l,\, \cos\theta_l,\, \cos\theta_\pi, \phi)$.

The principal results of the theoretical model discussed in [2,3] are as follows:

1. Branching ratio: This was calculated to be [2]

$$
\begin{aligned}
BR(K_L \to \pi^+\pi^-e^+e^-) &= (1.3\times10^{-7})_{Br} \\
&\quad + (1.8\times10^{-7})_{M1} \\
&\quad + (0.04\times10^{-7})_{CR} \\
&\approx 3.1\times10^{-7},
\end{aligned}
\tag{14}
$$

which agrees well with the result $(3.32\pm0.14\pm0.28)\times10^{-7}$ measured in the KTeV experiment [1]. (A preliminary branching ratio 2.9×10^{-7} has been reported by NA48 [8]).

2. Asymmetry in ϕ distribution: The model predicts a distribution of the form

$$
\frac{d\Gamma}{d\phi} \sim 1 - (\Sigma_3\cos2\phi + \Sigma_1\sin2\phi) \tag{15}
$$

which is in complete analogy with the distribution given by Eq. (8) in the case of $K_L \to \pi^+\pi^-\gamma$. The last term is CP- and T-violating, and produces an asymmetry

$$
\mathcal{A} = \frac{\left(\int_0^{\pi/2} - \int_{\pi/2}^{\pi} + \int_{\pi}^{3\pi/2} - \int_{3\pi/2}^{2\pi}\right)\frac{d\Gamma}{d\phi}d\phi}{\left(\int_0^{\pi/2} + \int_{\pi/2}^{\pi} + \int_{\pi}^{3\pi/2} + \int_{3\pi/2}^{2\pi}\right)\frac{d\Gamma}{d\phi}d\phi} = -\frac{2}{\pi}\Sigma_1. \tag{16}
$$

The predicted value [2,3] is

$$
|\mathcal{A}| = 15\%\,\sin(\phi_{+-} + \delta_0({M_K}^2) - \overline{\delta}_1) \approx 14\% \tag{17}
$$

to be compared with the KTeV result [1]

$$
|\mathcal{A}|_{KTeV} = (13.6\pm2.5\pm1.2)\% \tag{18}
$$

The parameters Σ_3 and Σ_1 are calculated to be $\Sigma_3 = -0.133$, $\Sigma_1 = 0.23$. The ϕ-distribution measured by KTeV agrees with this expectation (after acceptance corrections made in accordance with the model). It should be noted that the sign of Σ_1 (and of the asymmetry $\mathcal{A}$) depends on whether the numerical coefficient in g_{M1} is taken to be $+0.76$ or -0.76. The data support the positive sign chosen in Eq. (2).

3. Variation of $\Sigma_{1,3}$ with s_π: As shown in Fig. 2, the parameters Σ_1 and Σ_3 have a variation with s_π that is in close correspondence with the variation of S_1 and S_3. (Recall that the photon energy ω in $K_L \to \pi^+\pi^-\gamma$ can be expressed in terms of s_π: $s_\pi = {M_K}^2 - 2M_K\omega$.) In particular the zero of Σ_3 and the zero of S_3 occur at almost the same value of s_π. The similarity in the shape of Σ_1 and S_1 confirms the assertion that the asymmetry seen in $K_L \to \pi^+\pi^-e^+e^-$ is related to the CP-odd, T-odd component of the Stokes vector in $K_L \to \pi^+\pi^-\gamma$. The difference in scale is a measure of the analyzing power of the Dalitz pair process, viewed as a probe of the photon polarization.

Finally, we remark that our analysis takes for granted the validity of CPT invariance in the decays $K_L \to \pi^+\pi^-\gamma$ and $K_L \to \pi^+\pi^-e^+e^-$. If the assumption of CPT invariance is relaxed, the asymmetry observed in the KTeV experiment may be interpreted as some combination of T- and CPT-violation [9]. From the point of view of the present paper, the effect is understandable in a CPT-invariant framework, and follows inexorably from the empirical features of the decays $K_{L,S} \to \pi^+\pi^-\gamma$ mentioned at the outset.

Some of the ideas of this paper were presented by L. M. S. at the Kaon 99 Conference in Chicago [10].

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[1] KTeV collaboration, A. Alavi-Harati *et al.*, hep-ex/9908020, submitted to Phys. Rev. Lett.; A. Ledovskoy, talk at the Kaon 99 Conference, to be published in the Proceedings.

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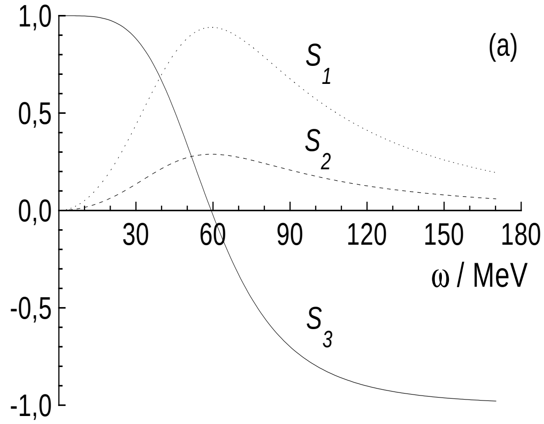
[7] In our numerical work we take $\phi_{+-} = 43^\circ$, $\delta_0({M_K}^2) = 40^\circ$, and an average value $\overline{\delta}_1 = 10^\circ$. We have also carried out a calculation which includes the s-dependence

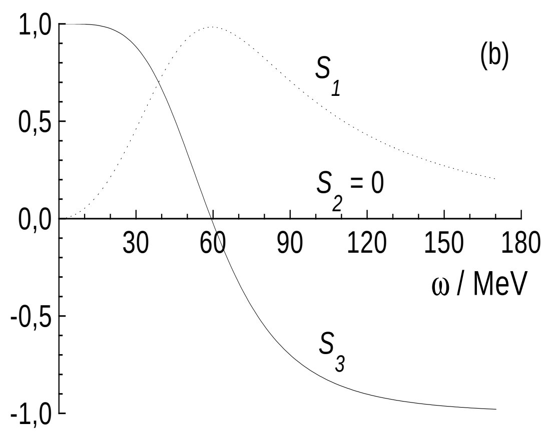
of $\delta_1(s)$, as well as the measured form factor $g_{M1}(s)$ [1,4] in place of the constant value $g_{M1} = 0.76$. The curves in Fig. 1a are mildly affected, the zero in S_3 and the maximum in S_1, S_2 shifting to about $50\,MeV$.

[8] NA48 collaboration, S. Wronka, talk at the Kaon 99 Conference, to be published in the Proceedings.

[9] L. Wolfenstein, Phys. Rev. Lett. **83**, 911 (1999); L. Alvarez-Gaume *et al.*, hep-ph/9903458; J. Ellis and N. E. Mavromatos, hep-ph/9903386; I. I. Bigi and A. I. Sanda, hep-ph/9904484.

[10] L. M. Sehgal, hep-ph/9908338, talk at the Kaon 99 Conference, to be published in the Proceedings.





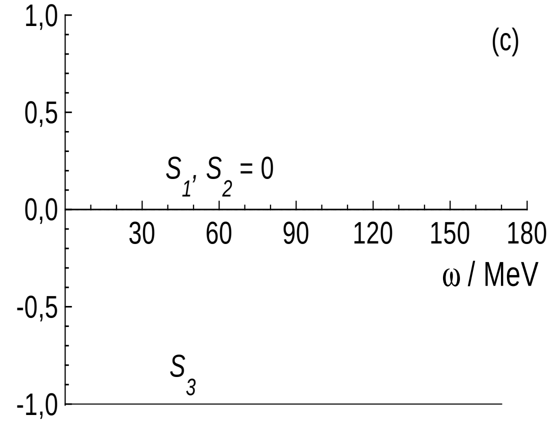


FIG. 1. (a) Stokes parameters of photon in $K_L \to \pi^+\pi^-\gamma$; (b) Hermitian limit $\delta_0 = \delta_1 = 0$, $arg\,\epsilon = \pi/2$; (c) CP-invariant limit $\epsilon \to 0$.

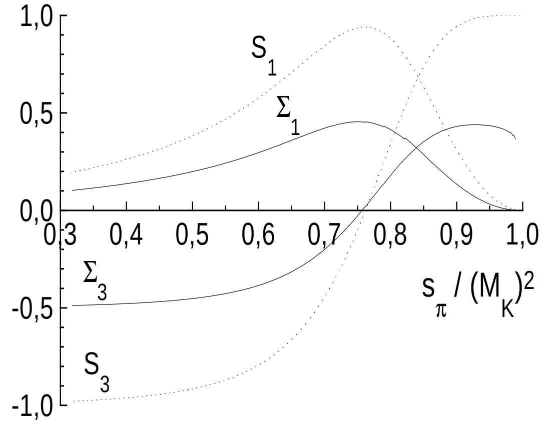


FIG. 2. Parameters Σ_1 and Σ_3 describing the ϕ - distribution in $K_L \to \pi^+\pi^-e^+e^-$, compared with the Stokes parameters S_1 and S_3 in $K_L \to \pi^+\pi^-\gamma$.